

k -CORDIALITY OF PATH AND CYCLE RELATED GRAPHS

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ABSTRACT

We discuss here k -cordial labeling of some path and cycle related graphs. We prove that the graph P_n^2 is k -cordial for all k and the pan graph C_n^{+1} is k -cordial for all even k .

KEYWORDS: Abelian Group; k -Cordial Labeling; Square Graph; Pan Graph

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INTRODUCTION

Throughout this work, by a graph we mean finite, connected, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$.

Definition 1.1. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices(edges) then the labeling is called a *vertex labeling*(an *edge labeling*).

A latest survey on various graph labeling problems can be found in Gallian[1].

Definition 1.2. Let $\langle A, * \rangle$ be any Abelian group. A graph $G = (V(G), E(G))$ is said to be *A-cordial* if there is a mapping $f: V(G) \rightarrow A$ which satisfies the following two conditions when the edge $e = uv$ is labeled as $f(u) * f(v)$

1. $|v_f(a) - v_f(b)| \leq 1$; for all $a, b \in A$,
2. $|e_f(a) - e_f(b)| \leq 1$; for all $a, b \in A$,

Where,

$v_f(a)$ =The number of vertices with label a ;

$v_f(b)$ =The number of vertices with label b ;

$e_f(a)$ =the number of edges with label a ;

$e_f(b)$ =the number of edges with label b .

We note that if $A = \langle \mathbb{Z}_k, +_k \rangle$ that is additive group of modulo k then the labeling is known as

k -cordial labeling.

The concept of k -cordial labeling was introduced by Hovey[3] and proved the following results.

- All the connected graphs are 3-cordial.
- All the trees are 3-cordial and 4-cordial.
- Cycles are k -cordial for all odd k .

Youssef [10] obtained the following results.

- C_{2k+1}^{+1} is not $(2k + 1)$ -cordial for $k \geq 1$.
- K_n is 4-cordial $\Leftrightarrow n \leq 6$.
- C_n^{+2} is 4-cordial $\Leftrightarrow n \not\equiv 2 \pmod{4}$.
- $K_{m,n}$ is 4-cordial $\Leftrightarrow m \text{ or } n \not\equiv 2 \pmod{4}$.

In [4, 5] Kanani and Modha proved various results related to 5-cordial and 7-cordial labeling. They also proved k -cordial labeling of fan in [6].

In [7,8] Kanani and Rathod derived some families of 4-cordial graphs.

In this paper we consider the following definitions of standard graphs.

- Let G be a simple connected graph. The *square of graph G* denoted by G^2 is defined to be the graph with the same vertex set as G and in which two vertices u and v are joined by an edge $\Leftrightarrow$ in G we have
- $1 \leq d(u, v) \leq 2$.
- The n -pan graph is the graph obtained by joining a cycle graph C_n to a singleton graph K_1 with a bridge.

For any undefined term in graph theory we rely upon Gross and Yellen[2].

MAIN RESULTS

Theorem 2.1. The Graph P_n^2 is k -cordial.

Proof: Let P_n^2 be the square graph of path P_n with vertices $v_1, v_2, \dots, v_n$. We note that $|V(G)| = n$ and $|E(G)| = 2n - 3$.

We define k -cordial labeling $f: V(G) \rightarrow \mathbb{Z}_k$ as follows.

$$f(v_i) = p_i - 1; i \equiv p_i \pmod{k}, 1 \leq i \leq n,$$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k -cordial labeling. That is, the graph P_n^2 is

k -cordial for all k .

Illustration 2.2. The square graph P_{11}^2 and its 8-cordial labeling is shown in Figure 1.

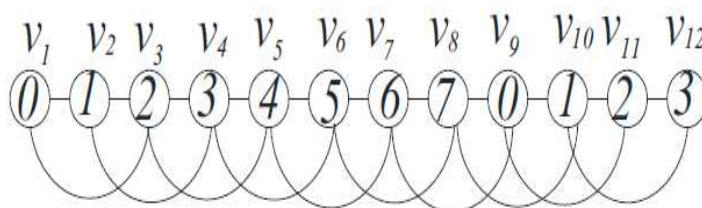


Figure 1: 8-Cordial Labeling of P_{12}^2 .

Theorem 2.3. C_n^{+1} is k -cordial for all even k and $n = k + j$, $0 \leq j \leq k - 1$.

Proof: Let $G = C_n^{+1}$ be the cycle with one pendant edge known as Pan graph. Let $n = k + j$, where

$0 \leq j \leq k - 1$. We divide the n vertices of cycle of the pan C_n^{+1} into two blocks of k and j vertices namely $v_1, v_2, \dots, v_k$ and $v'_1, v'_2, \dots, v'_j$. Let v' be the pendant vertex. We note that $|V(G)| = n + 1$ and

$$|E(G)| = n + 1.$$

To define k -cordial labeling $f: V(G) \rightarrow \mathbb{Z}_k$ we consider the following cases.

Case 1: $j = 0$

For the first block of k vertices,

$$\begin{aligned} f(v_i) &= k - \frac{p_i + 1}{2}; & i \equiv p_i \pmod{k}, & \quad p_i \text{ is odd,} \\ &= \frac{k}{2} - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & \quad p_i \text{ is even, } 1 \leq i \leq k, \\ f(v') &= \frac{k}{2} - 1. \end{aligned}$$

Case 2: $j = 1$

For the first block of k vertices,

$$\begin{aligned} f(v_i) &= k - \frac{p_i + 1}{2}; & i \equiv p_i \pmod{k}, & \quad p_i \text{ is odd,} \\ &= \frac{k}{2} - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & \quad p_i \text{ is even, } 1 \leq i \leq k, \end{aligned}$$

The labeling pattern of second block of j vertex is

$$\begin{aligned} f(v'_1) &= \frac{k}{2} - 1, \\ f(v') &= \frac{k}{2} - 2. \end{aligned}$$

Case 3: $2 \leq j \leq k - 2$

For the first block of k vertices,

$$\begin{aligned} f(v_i) &= k - \frac{p_i + 1}{2}; & i \equiv p_i \pmod{k}, & \quad p_i \text{ is odd,} \\ &= \frac{k}{2} - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & \quad p_i \text{ is even, } 1 \leq i \leq k, \end{aligned}$$

The labeling pattern of second block of j vertices, where $2 \leq j \leq k-2$ is divided into following subcases.

Subcase 1: If j is odd.

Sub-subcase 1: If $\frac{j+1}{2}$ is odd.

$$\begin{aligned} f(v'_i) &= \frac{k}{2} - \frac{i+1}{2}; & i \text{ is odd,} \\ &= k - \frac{i}{2}; & i \text{ is even, } 1 \leq i < \frac{j+1}{2}, \\ f(v'_i) &= k - \frac{i+1}{2}; & i \text{ is odd,} \\ &= \frac{k}{2} - \frac{i}{2}; & i \text{ is even, } \frac{j+1}{2} \leq i \leq k-2, \\ f(v') &= \frac{k}{2} - \frac{j+1}{2}. \end{aligned}$$

Sub-subcase 2: If $\frac{j+1}{2}$ is even.

$$\begin{aligned} f(v'_i) &= \frac{k}{2} - \frac{i+1}{2}; & i \text{ is odd,} \\ &= k - \frac{i}{2}; & i \text{ is even, } 1 \leq i < \frac{j+1}{2}, \\ f(v'_i) &= k - \frac{i-1}{2}; & i \text{ is odd,} \\ &= \frac{k}{2} - \frac{i+2}{2}; & i \text{ is even, } \frac{j+1}{2} \leq i \leq k-2, \\ f(v') &= \frac{k}{2} - \frac{j+3}{2}. \end{aligned}$$

Subcase 2: If j is even.

$$\begin{aligned} f(v'_i) &= \frac{k}{2} - \frac{i+1}{2}; & i \text{ is odd,} \\ &= k - \frac{i}{2}; & i \text{ is even, } 1 \leq i \leq j, \\ f(v') &= \frac{k}{2} - \frac{j+2}{2}. \end{aligned}$$

Case 4: $j = k-1$

In this case divide the $2k-1$ vertices $v_1, v_2, \dots, v_k, v'_1, v'_2, \dots, v'_{k-1}$ of the cycle C_n into $k-1$ blocks of two vertices and one block of one vertex. Next label the i^{th} block of the cycle with i for $1 \leq i \leq k-1$ and last block with one vertex by 0 and the vertex v' by 0 .

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k -cordial labeling. Hence, C_n^{+1} is

k -cordial for all even k and $n = k + j$, $0 \leq j \leq k-1$.

Illustration.2.4. The Pan graph C_{25}^{+1} and its 20-cordial labeling is shown in Figure 2.

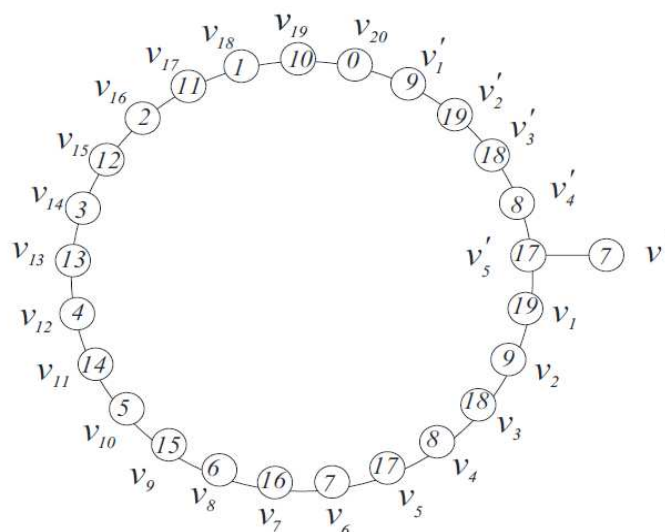


Figure 2: 20-Cordial Labeling of Pan Graph C_{25}^{+1}

Theorem 2.5. C_n^{+1} is k -cordial for all even k and $n = 2tk + j$, where $t \in \mathbb{N} \cup \{0\}$ and $0 \leq j \leq k - 1$.

Proof: Let $G = C_n^{+1}$ be the cycle with one pendant edge known as Pan graph. Let $n = 2tk + j$, where

$0 \leq j \leq k - 1$ and $t \geq 0$. We divide the n cycle vertices of the pan C_n^{+1} into two blocks of $2tk$ and j vertices namely $v_1, v_2, \dots, v_{2tk}$ and $v'_1, v'_2, \dots, v'_j$. Let v' be the pendant vertex. We note that $|V(G)| = n + 1$ and

$$|E(G)| = n + 1.$$

To define k -cordial labeling $f: V(G) \rightarrow \mathbb{Z}_k$ we consider the following cases.

Case 1: $j = 0$

For the first block of $2tk$ vertices, where $t \geq 1$,

$$\begin{aligned} f(v_i) &= k - \frac{2i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd,} \\ &= \frac{k}{2} - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k + 1 \leq i \leq mk, \text{ } m \text{ is odd, } 1 \leq m \leq 2t, \\ f(v_i) &= \frac{k}{2} - \frac{2i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd, } t \text{ is even,} \\ &= k - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k + 1 \leq i \leq mk, \text{ } m \text{ is even, } 1 \leq m \leq 2t, \\ f(v') &= k - 1, \end{aligned}$$

Case 2: $j = 1$

For the first block of $2tk$ vertices, where $t \geq 1$,

$$\begin{aligned} f(v_i) &= k - \frac{2i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd,} \\ &= \frac{k}{2} - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k + 1 \leq i \leq mk, \text{ } m \text{ is odd, } 1 \leq m \leq 2t, \end{aligned}$$

$$\begin{aligned}
 f(v_i) &= \frac{k}{2} - \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd, } t \text{ is even,} \\
 &= k - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k+1 \leq i \leq mk, \text{ } m \text{ is even, } 1 \leq m \leq 2t,
 \end{aligned}$$

The labeling pattern of second block of f vertex is,

$$f(v'_1) = k - 1,$$

$$f(v') = \frac{k}{2} - 1.$$

Case 3: $f = 2$

For the first block of $2tk$ vertices, where $t \geq 1$,

$$\begin{aligned}
 f(v_i) &= k - \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd,} \\
 &= \frac{k}{2} - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k+1 \leq i \leq mk, \text{ } m \text{ is odd, } 1 \leq m \leq 2t, \\
 f(v_i) &= \frac{k}{2} - \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd, } t \text{ is even,} \\
 &= k - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k+1 \leq i \leq mk, \text{ } m \text{ is even, } 1 \leq m \leq 2t,
 \end{aligned}$$

The labeling pattern of second block of f vertex is,

$$f(v'_1) = \frac{k}{2} - 1$$

$$f(v'_2) = k - 1,$$

$$f(v') = \frac{k}{2}.$$

Case 4: $3 \leq f \leq k-1$

For the first block of $2tk$ vertices, where $t \geq 1$,

$$\begin{aligned}
 f(v_i) &= k - \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd,} \\
 &= \frac{k}{2} - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k+1 \leq i \leq mk, \text{ } m \text{ is odd, } 1 \leq m \leq 2t, \\
 f(v_i) &= \frac{k}{2} - \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd, } t \text{ is even,} \\
 &= k - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k+1 \leq i \leq mk, \text{ } m \text{ is even, } 1 \leq m \leq 2t,
 \end{aligned}$$

The labeling pattern of second block of f vertices, where $3 \leq f \leq k-1$ is divided into following two subcases.

Subcase 1: If f is odd.

$$\begin{aligned}
 f(v'_i) &= k - \frac{i+1}{2}; & i \text{ is odd,} \\
 &= \frac{k}{2} - \frac{i}{2}; & i \text{ is even, } 3 \leq i \leq k-1,
 \end{aligned}$$

$$f(v') = \frac{k}{2} - \frac{j+1}{2}.$$

Subcase 2: If j is even.

For $3 \leq j \leq \frac{k}{2}$,

$$\begin{aligned} f(v'_i) &= k - \frac{i+1}{2}; & i \text{ is odd,} \\ &= \frac{k}{2} - \frac{i}{2}; & i \text{ is even,} \end{aligned} \quad 1 \leq i < j$$

$$f(v'_i) = k - \frac{i+2}{2}; \quad \text{if } i = j,$$

$$f(v') = \frac{k}{2} - \frac{j}{2}.$$

For $\frac{k}{2} < j \leq k-1$

The labeling pattern of remaining block of j vertices is divided into following two sub-subcases.

Sub-subcase 1: If $\frac{j}{2}$ is odd.

$$\begin{aligned} f(v'_i) &= k - \frac{i+1}{2}; & i \text{ is odd,} \\ &= \frac{k}{2} - \frac{i}{2}; & i \text{ is even,} \end{aligned} \quad 1 \leq i < \frac{j}{2}$$

$$\begin{aligned} f(v'_i) &= \frac{k}{2} - \frac{i+1}{2}; & i \text{ is odd,} \\ &= k - \frac{i}{2}; & i \text{ is even,} \end{aligned} \quad \frac{j}{2} \leq i \leq k-1$$

$$f(v') = \frac{k}{2} - \frac{j+2}{2}.$$

Sub-subcase 2: If $\frac{j}{2}$ is even.

$$\begin{aligned} f(v'_i) &= k - \frac{i+1}{2}; & i \text{ is odd,} \\ &= \frac{k}{2} - \frac{i}{2}; & i \text{ is even,} \end{aligned} \quad 1 \leq i < \frac{j}{2}$$

$$\begin{aligned} f(v'_i) &= \frac{k}{2} - \frac{i-1}{2}; & i \text{ is odd,} \\ &= k - \frac{i+2}{2}; & i \text{ is even,} \end{aligned} \quad \frac{j}{2} \leq i \leq k-1$$

$$f(v') = \frac{k}{2} - \frac{j}{2}.$$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k -cordial labeling. Hence, C_n^{+k} is k -cordial for all even k and $n = 2tk + j$, where $t \geq 0$ and $0 \leq j \leq k-1$.

Illustration 2.6. The Pan graph C_{12}^{+4} and its 12-cordial labeling is shown in Figure 3.

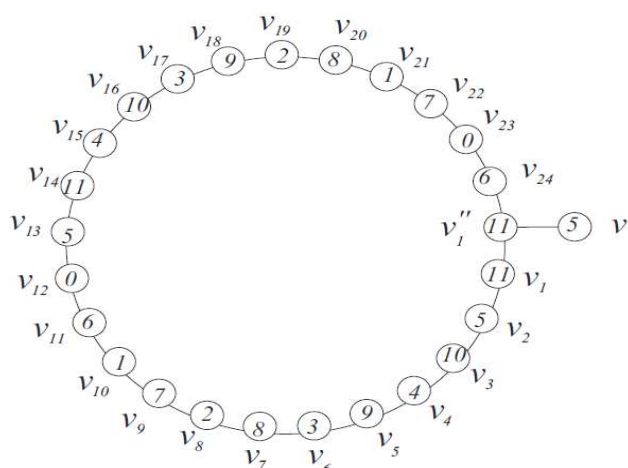


Figure 3: 12-Cordial Labeling Of Pan Graph C_{24}^{+1}

Theorem 2.7. C_n^{+1} is k -cordial for all even k and $n = 2tk + k + j$, where $t \in \mathbb{N}$ and $0 \leq j \leq k - 1$.

Proof: Let $G = C_n^{+1}$ be the cycle with one pendant edge known as Pan graph. Let $n = 2tk + k + j$, where

$0 \leq j \leq k - 1$ and $t \geq 1$. We divide the n cycle vertices of the pan C_n^{+1} into three blocks of $2tk$, k and j vertices namely $v_1, v_2, \dots, v_{2tk}, v_1', v_2', \dots, v_k'$ and $v_1'', v_2'', \dots, v_j''$. Let v' be the pendant vertex. We note that $|V(G)| = n + 1$ and $|E(G)| = n + 1$.

Define k -cordial labeling $f: V(G) \rightarrow \mathbb{Z}_k$ as follows

For the first block of $2tk$ vertices, where $t \geq 1$,

$$\begin{aligned} f(v_i) &= k - \frac{p_i + 1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd,} \\ &= \frac{k}{2} - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k + 1 \leq i \leq mk, \text{ } m \text{ is odd, } 1 \leq m \leq 2t, \\ f(v_i) &= \frac{k}{2} - \frac{p_i + 1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd, } t \text{ is even,} \\ &= k - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k + 1 \leq i \leq mk, \text{ } m \text{ is even, } 1 \leq m \leq 2t, \\ f(v') &= k - 1. \end{aligned}$$

For the second block of k vertices,

$$\begin{aligned} f(v_i') &= k - \frac{p_i' + 1}{2}; & i \equiv p_i' \pmod{k}, & & p_i' \text{ is odd,} \\ &= \frac{k}{2} - \frac{p_i'}{2}; & i \equiv p_i' \pmod{k}, & & p_i' \text{ is even, } 1 \leq p_i' \leq k, \\ f(v'') &= \frac{k}{2} - 1. \end{aligned}$$

The labeling pattern for the third block of j vertices is divided into following cases.

Case 1: $j = 0$

For the first block of $2tk$ vertices, where $t \geq 1$,

$$\begin{aligned}
 f(v_i) &= k - \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd,} \\
 &= \frac{k}{2} - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k+1 \leq i \leq mk, \text{ } m \text{ is odd, } 1 \leq m \leq 2t, \\
 f(v_i) &= \frac{k}{2} - \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd, } t \text{ is even,} \\
 &= k - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k+1 \leq i \leq mk, \text{ } m \text{ is even, } 1 \leq m \leq 2t,
 \end{aligned}$$

For the second block of k vertices,

$$\begin{aligned}
 f(v'_i) &= k - \frac{p'_i+1}{2}; & i \equiv p'_i \pmod{k}, & & p'_i \text{ is odd,} \\
 &= \frac{k}{2} - \frac{p'_i}{2}; & i \equiv p'_i \pmod{k}, & & p'_i \text{ is even, } 1 \leq p'_i \leq k, \\
 f(v') &= \frac{k}{2} - 1
 \end{aligned}$$

Case 2: $j = 1$

For the first block of $2tk$ vertices, where $t \geq 1$,

$$\begin{aligned}
 f(v_i) &= k - \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd,} \\
 &= \frac{k}{2} - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k+1 \leq i \leq mk, \text{ } m \text{ is odd, } 1 \leq m \leq 2t, \\
 f(v_i) &= \frac{k}{2} - \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd, } t \text{ is even,} \\
 &= k - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k+1 \leq i \leq mk, \text{ } m \text{ is even, } 1 \leq m \leq 2t,
 \end{aligned}$$

For the second block of k vertices,

$$\begin{aligned}
 f(v'_i) &= k - \frac{p'_i+1}{2}; & i \equiv p'_i \pmod{k}, & & p'_i \text{ is odd,} \\
 &= \frac{k}{2} - \frac{p'_i}{2}; & i \equiv p'_i \pmod{k}, & & p'_i \text{ is even, } 1 \leq p'_i \leq k,
 \end{aligned}$$

The labeling pattern of third block of j vertex is

$$\begin{aligned}
 f(v''_1) &= \frac{k}{2} - 1, \\
 f(v') &= \frac{k}{2} - 2.
 \end{aligned}$$

Case 3: $2 \leq j \leq k-2$

For the first block of $2tk$ vertices, where $t \geq 1$,

$$\begin{aligned}
 f(v_i) &= k - \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd,} \\
 &= \frac{k}{2} - \frac{p_i}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is even, } (m-1)k+1 \leq i \leq mk, \text{ } m \text{ is odd, } 1 \leq m \leq 2t, \\
 f(v_i) &= \frac{k}{2} - \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, & & p_i \text{ is odd, } t \text{ is even,}
 \end{aligned}$$

$$= k - \frac{p_i}{2}; \quad i \equiv p_i \pmod{k}, \quad p_i \text{ is even, } (m-1)k+1 \leq i \leq mk, \quad m \text{ is even, } 1 \leq m \leq 2t,$$

For the second block of k vertices,

$$f(v_i') = k - \frac{p_i'+1}{2}; \quad i \equiv p_i' \pmod{k}, \quad p_i' \text{ is odd,}$$

$$= \frac{k}{2} - \frac{p_i'}{2}; \quad i \equiv p_i' \pmod{k}, \quad p_i' \text{ is even, } 1 \leq p_i' \leq k,$$

The labeling pattern of third block of j vertices, where $2 \leq j \leq k-2$ is divided into following sub cases.

Subcase 1: If j is odd.

Sub-subcase 1: If $\frac{j+1}{2}$ is odd.

$$f(v_i'') = \frac{k}{2} - \frac{i+1}{2}; \quad i \text{ is odd,}$$

$$= k - \frac{i}{2}; \quad i \text{ is even, } 1 \leq i \leq \frac{j+1}{2},$$

$$f(v_i'') = k - \frac{i+1}{2}; \quad i \text{ is odd,}$$

$$= \frac{k}{2} - \frac{i}{2}; \quad i \text{ is even, } \frac{j+1}{2} \leq i \leq k-2,$$

$$f(v') = \frac{k}{2} - \frac{j+1}{2}.$$

Sub-subcase 2: If $\frac{j+1}{2}$ is even.

$$f(v_i'') = \frac{k}{2} - \frac{i+1}{2}; \quad i \text{ is odd,}$$

$$= k - \frac{i}{2}; \quad i \text{ is even, } 1 \leq i \leq \frac{j+1}{2},$$

$$f(v_i'') = k - \frac{i-1}{2}; \quad i \text{ is odd,}$$

$$= \frac{k}{2} - \frac{i+2}{2}; \quad i \text{ is even, } \frac{j+1}{2} \leq i \leq k-2$$

$$f(v') = \frac{k}{2} - \frac{j+3}{2}.$$

Subcase 2: If j is even.

$$f(v_i'') = \frac{k}{2} - \frac{i+1}{2}; \quad i \text{ is odd,}$$

$$= k - \frac{i}{2}; \quad i \text{ is even, } 1 \leq i \leq j,$$

$$f(v') = \frac{k}{2} - \frac{j+2}{2}.$$

Case 4: $j = k-1$

In this case divide the $2tk + 2k - 1$ vertices $v_1, v_2, \dots, v_{2tk}, v_1', v_2', \dots, v_k', v_1'', v_2'', \dots, v_{k-1}''$ of the cycle C_n into $(tk + k) - 1$ blocks of two vertices and one block of one vertex. Next label the i^{th} block of the cycle with i for

$1 \leq i \leq k-1$ and last block with one vertex by 0 and the vertex v' by 0.

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k -cordial labeling. Hence, C_n^{+1} is

k -cordial for all even k and $n = 2tk + k + j$, where $0 \leq j \leq k-1$ and $t \geq 1$.

Illustration 2.8. The The Pan graph C_{15}^{+1} and its 8-cordial labeling is shown in Figure 4.

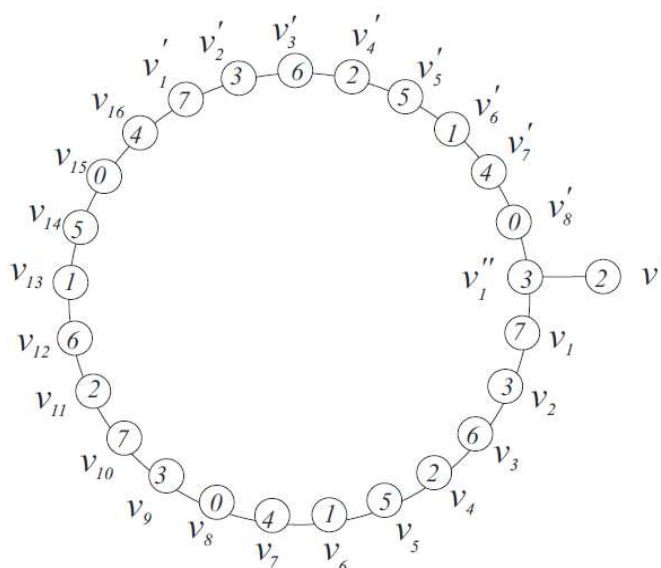


Figure 4: 8-Cordial Labeling Of Pan Graph C_{15}^{+1}

CONCLUSIONS

To investigate similar results for any integer k is an open problem.

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